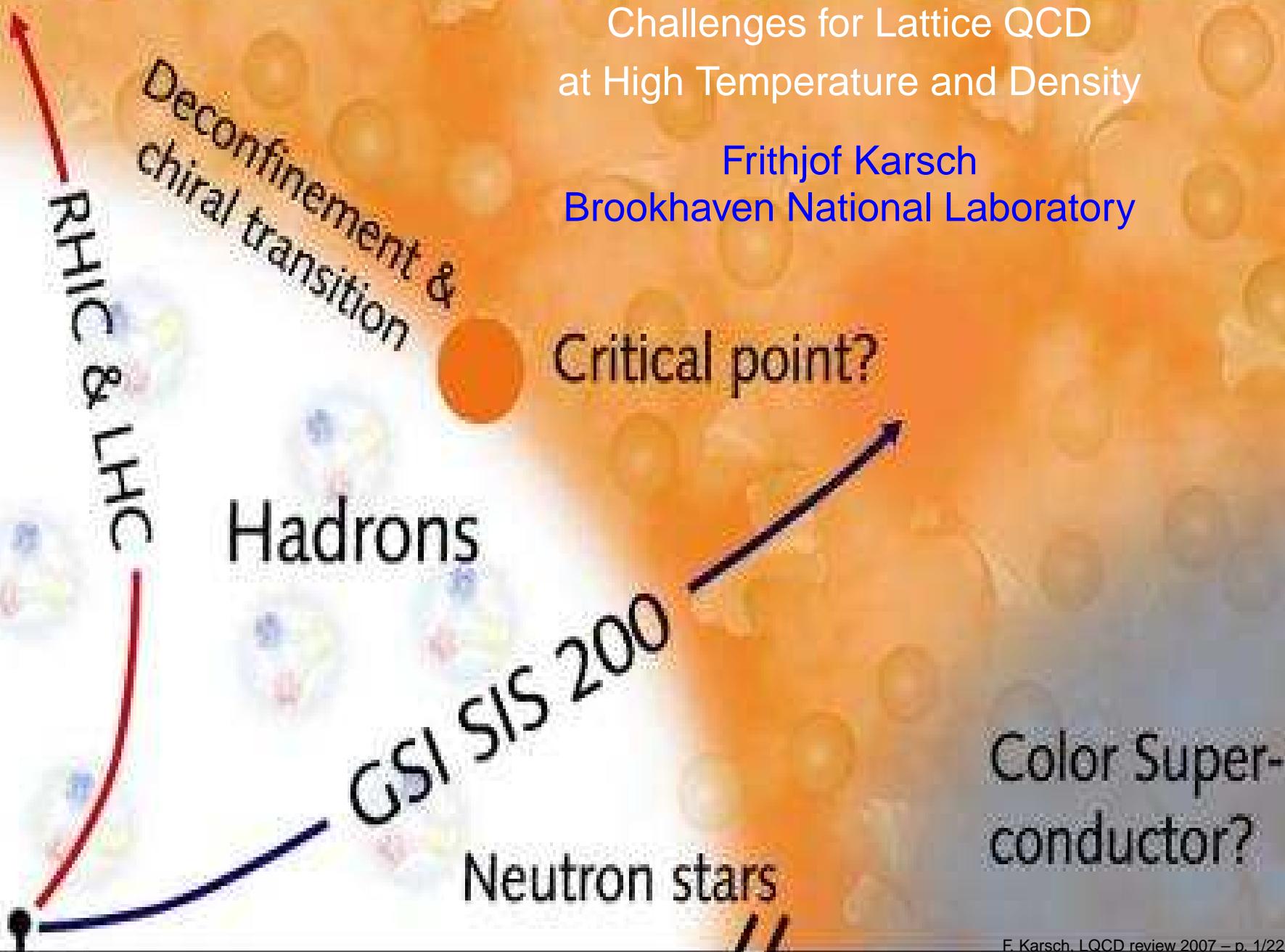


Quarks and Gluons

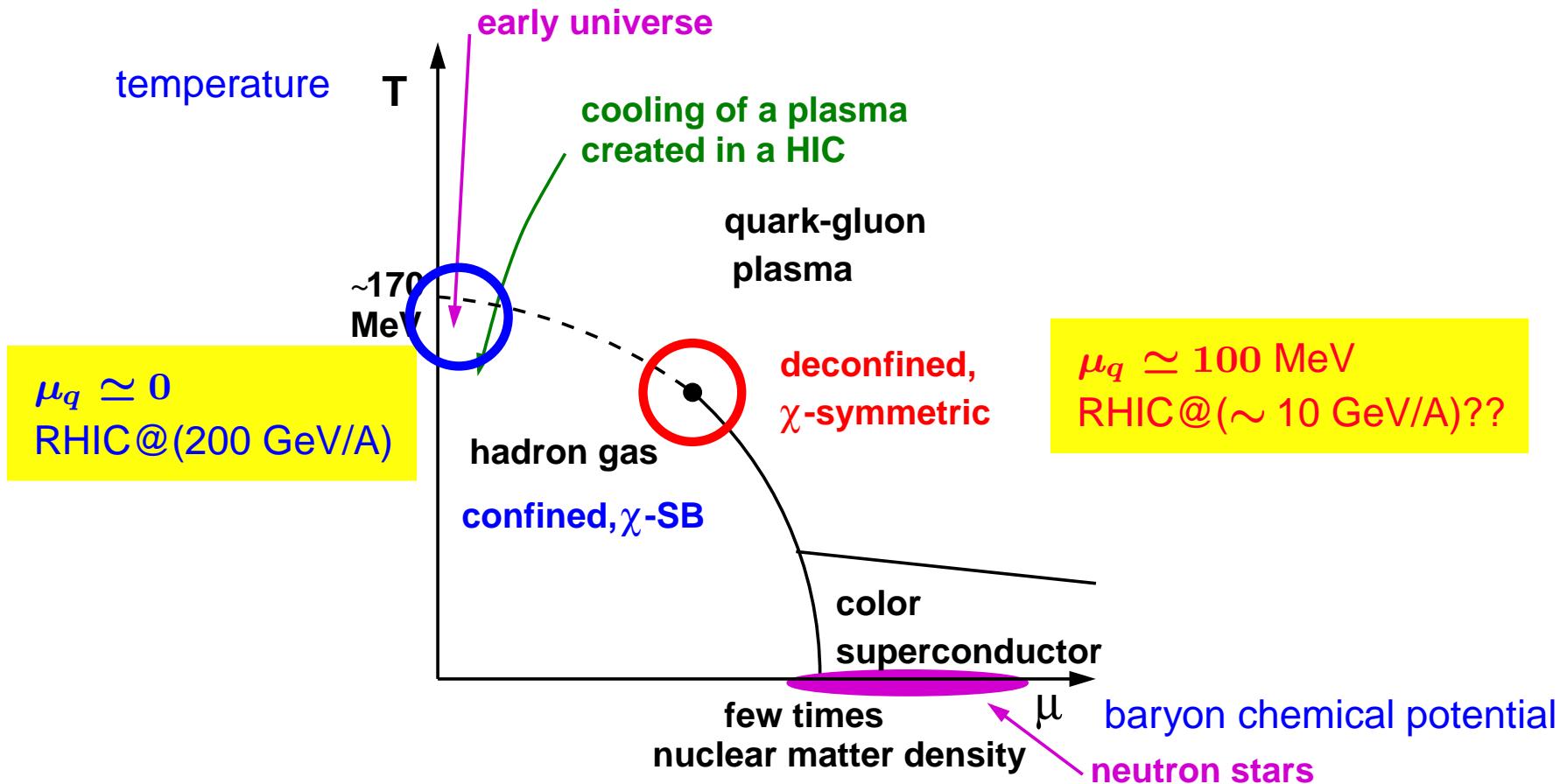
Challenges for Lattice QCD
at High Temperature and Density

Frithjof Karsch
Brookhaven National Laboratory



Phase diagram of strongly interacting matter

RHIC I/II & LHC \Leftrightarrow LGT at vanishing chemical potential
FAIR@GSI & RHIC at low energy \Leftrightarrow LGT at non zero chemical potential



Major LGT Thermodynamics projects

BNL-Columbia-RIKEN and Bielefeld:

- p4-improved staggered fermions, $\mu = 0$ and $\mu > 0$, T_c , EoS, phase diagram, $N_\tau = 4$ and **6**
QCDOC at BNL (**partly funded through the LQCD project**) and **apeNEXT** at Bielefeld

hotQCD: BNL-Columbia-RIKEN, MILC, LLNL, LANL:

- p4-improved and asqtad staggered fermions, $\mu = 0$ EoS and T_c , $N_\tau = 8$
BlueGene/L at Livermore

Budapest-Wuppertal:

- standard staggered fermions with stout smearing, EoS, T_c ; $N_\tau = 4 - 10$
PC-cluster ALICE, graphics cards

WHOT-QCD: Tokyo-Tsukuba-BNL:

- Wilson fermions, $\mu = 0$ and $\mu > 0$, T_c and phase diagram, $N_\tau = 4$ and **6**
BlueGene/L at KEK

European twisted mass collaboration (DESY/Zeuthen, Berlin, Münster, Rome,...):

- twisted mass Wilson fermions, T_c , $N_\tau = 8$?
apeNEXT at Zeuthen and Rome

Thermodynamics projects realized with LQCD project resources

- 2005-2006: T_c
QCD Thermodynamics with improved staggered fermions (BNL-Columbia-RIKEN)
 - M. Cheng, N. H. Christ, S. Datta, J. van der Heide, C. Jung, F. Karsch,
O. Kaczmarek, E. Laermann, R. D. Mawhinney, C. Miao, P. Petreczky, K. Petrov,
C. Schmidt, T. Umeda, Phys.Rev. D74 (2006) 054507
- 2006-2007: EoS, spatial screening lengths
QCD Thermodynamics with improved staggered fermions (continuation)
(BNL-Columbia-RIKEN)
 - M. Cheng et al, Phys. Rev. D75 (2007) 034506
 - M. Cheng et al, hep-lat/0612030, to be published in Eur. Phys. J. C
- 2007-2008: Non-zero μ , phase diagram
3-flavor QCD thermodynamics with improved staggered fermions
(BNL-Columbia-RIKEN)

QCD Thermodynamics 2005-2009: What do we want to know?

I. T_c , ϵ_c , EoS:

- basic input to hydrodynamic modeling of heavy ion collisions;
confront models with lattice calculations (resonance gas, quasi-particle gas, high-T pert. theory, HTL-resummation, AdS/CFT ...)
test universal aspects of deconfinement and chiral symmetry restoration
in 2, (2 + 1), 3-flavor QCD

II. search for the critical point at $\mu > 0$:

- chiral critical point: does it exist?; location of the chiral critical point;
direct evidence for 1st order regime;
location of the transition line for $\mu > 0$; density fluctuations; $T_c(\mu) \equiv T_{\text{freeze}}$?
-

III. In-medium properties of hadrons

- quarkonium spectroscopy and heavy quark diffusion
- light quark bound states and thermal dilepton/photon rates

QCD Thermodynamics 2005-2009: What do we want to know?

I. T_c , ϵ_c , EoS:

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Transition temperature

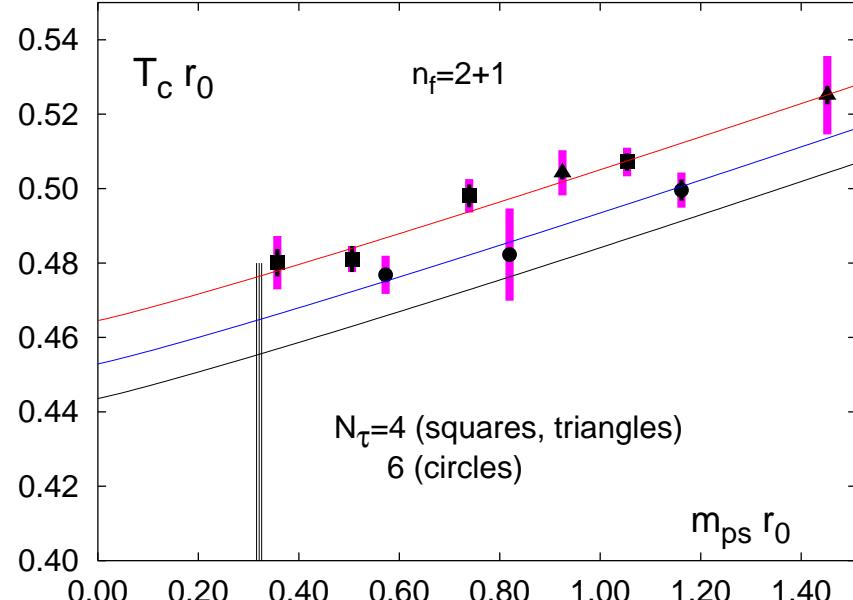
RBC-Bielefeld, Phys. Rev. D74, 054507 (2006)

Phys. Rev. D75, 034506 (2007)

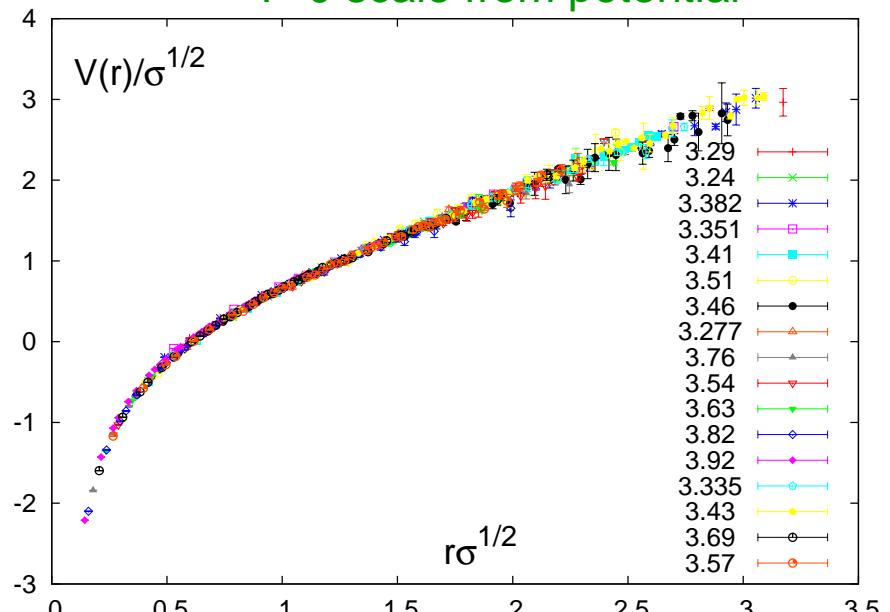
extrapolation to chiral and continuum limit

$$(r_0 T_c)_{N_\tau} = (r_0 T_c)_{cont.} + b (m_{PS} r_0)^d + c/N_\tau^2$$

RBC-Bielefeld



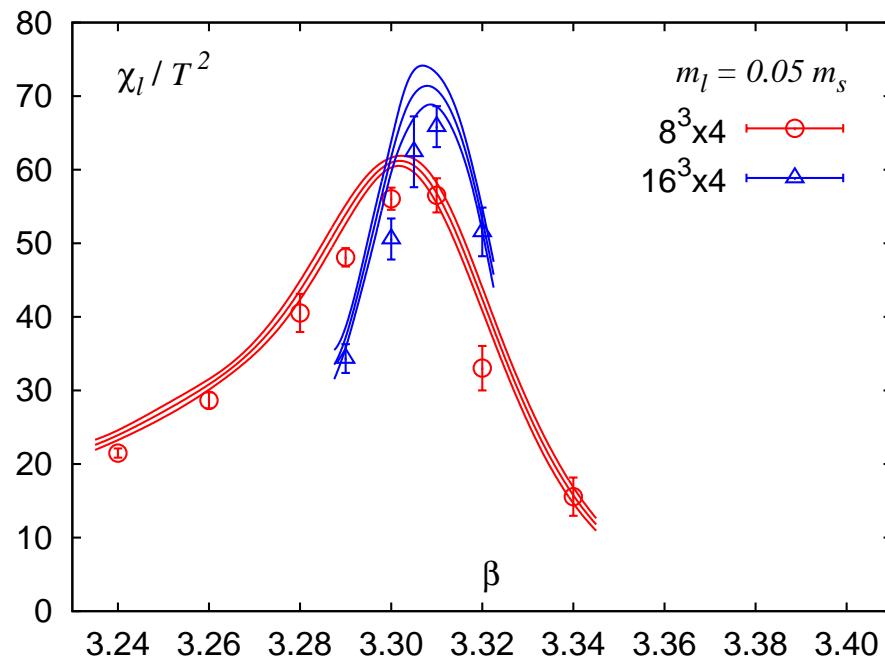
T=0 scale from potential



$\Rightarrow r_0 T_c = 0.456(7)^{+3}_{-1}$ at physical ($m_{u,d}, m_s$) point

$\Rightarrow T_c = 192(7)(4)$ MeV

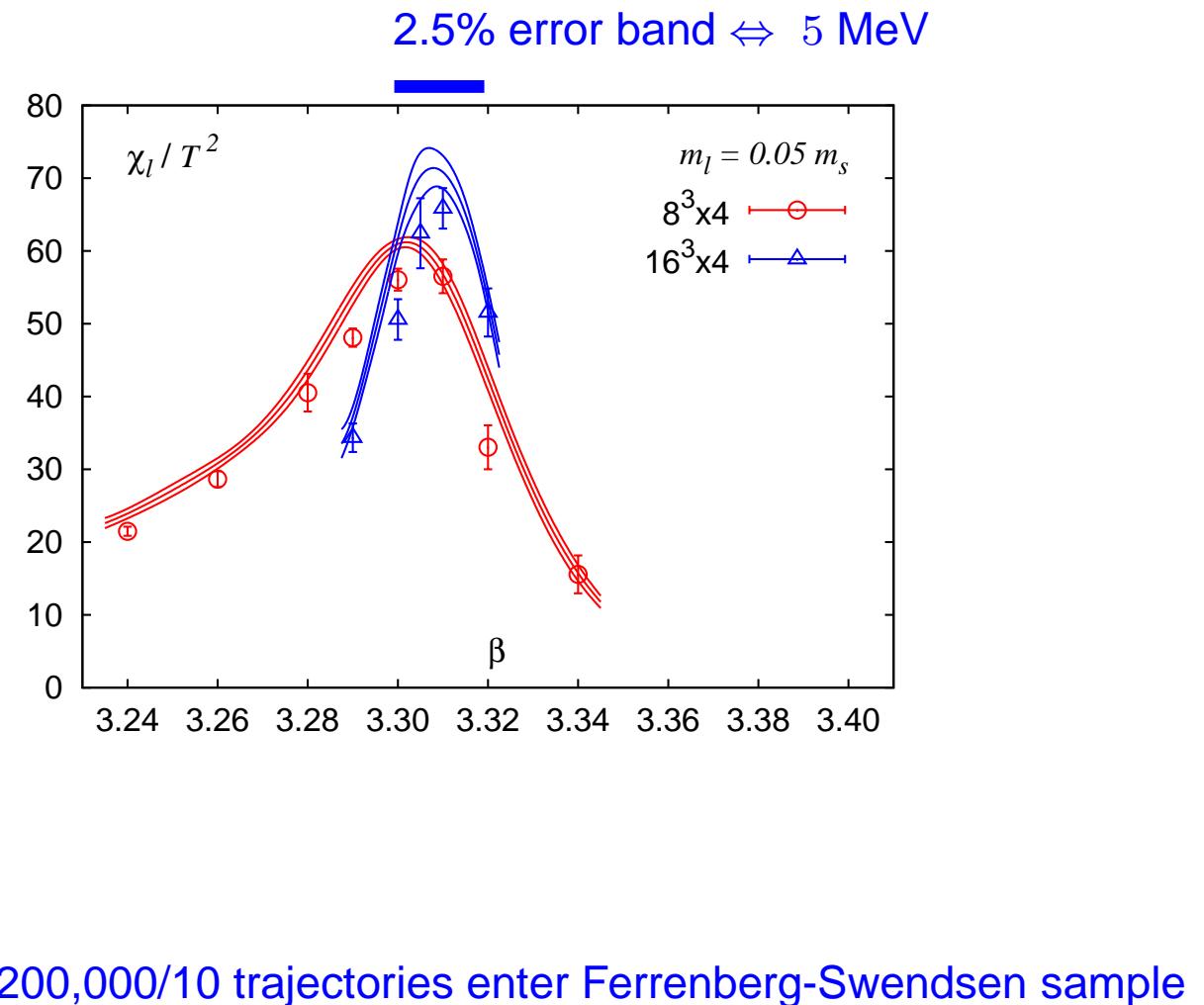
Chiral and L susceptibility, $N_\tau = 4$



Chiral and L susceptibility, $N_\tau = 4$

data sample for
smallest quark mass
on $16^3 \times 4$ lattice

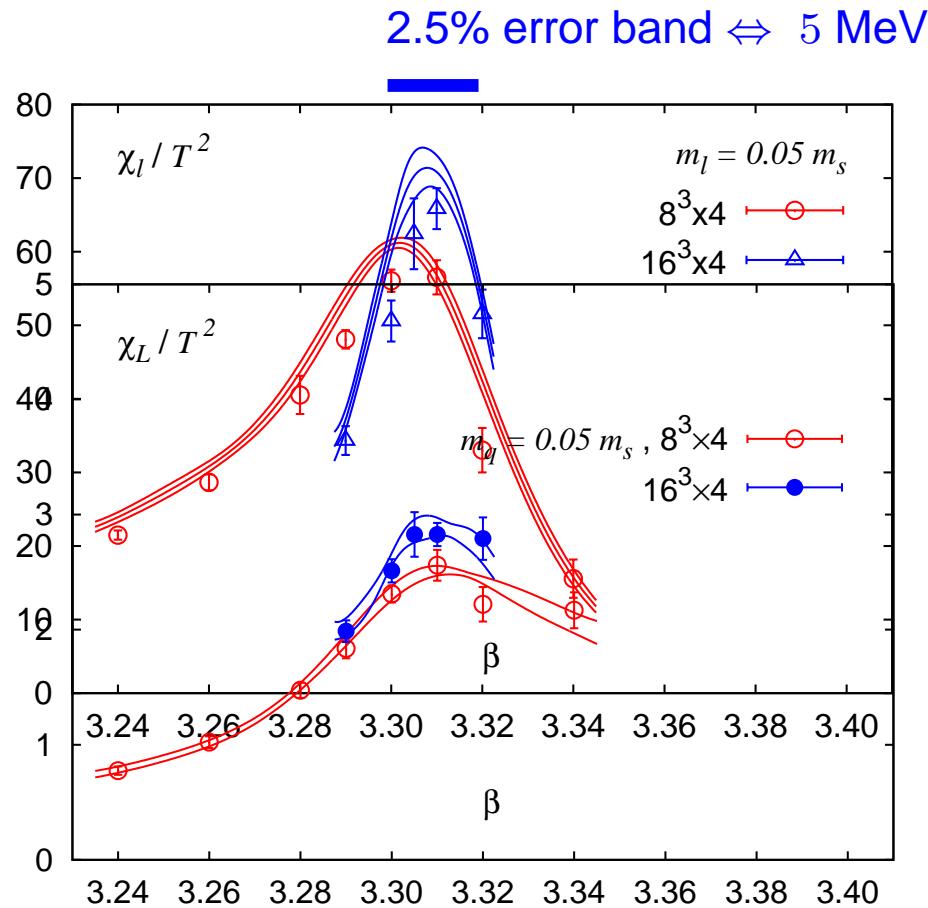
β	no. of conf.
3.2900	38960
3.3000	40570
3.3050	32950
3.3100	42300
3.3200	39050



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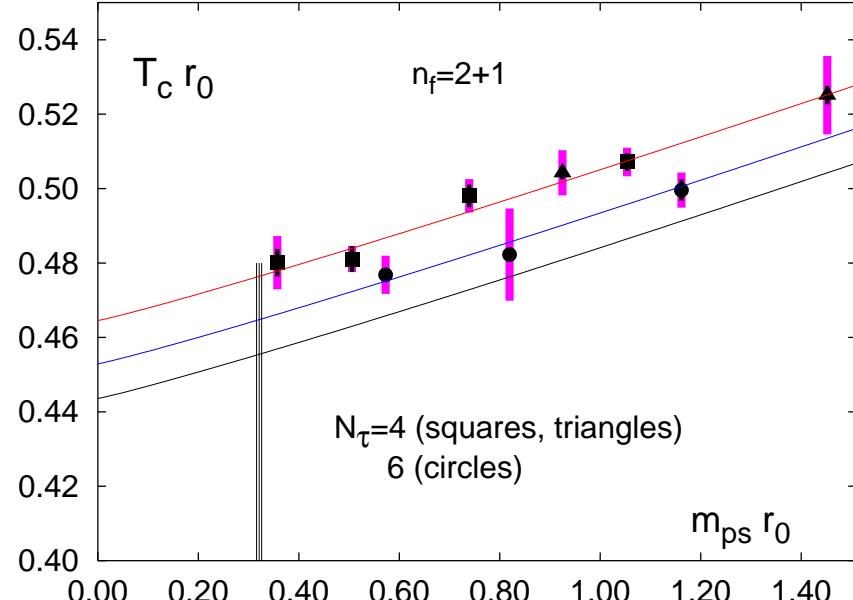
RBC-Bielefeld, Phys. Rev. D74, 054507 (2006)

Phys. Rev. D75, 034506 (2007)

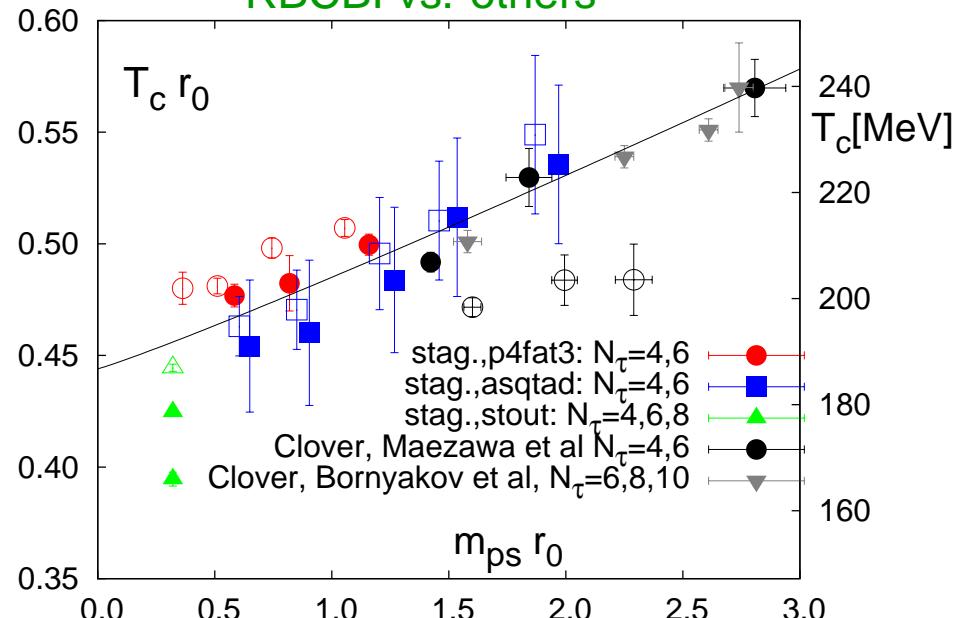
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RBC-Bielefeld



RBCBI vs. others

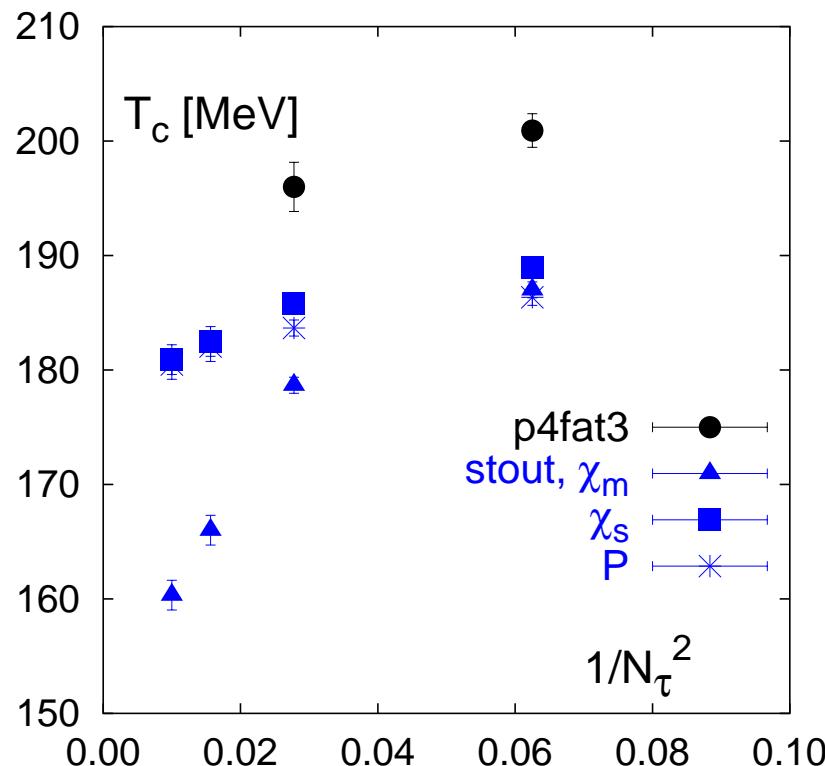


$\Rightarrow r_0 T_c = 0.456(7)^{+3}_{-1}$ at physical ($m_{u,d}, m_s$) point

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extrapolations to physical point

- RBC-Bielefeld (p4fat3 (p4)) vs. Wuppertal (stout (stand. staggered))
- results for $N_\tau = 4, 6$ differ by 15% but show similar cut-off dependence
- stout results for different observables no longer consistent with each other for $N_\tau = 8, 10$

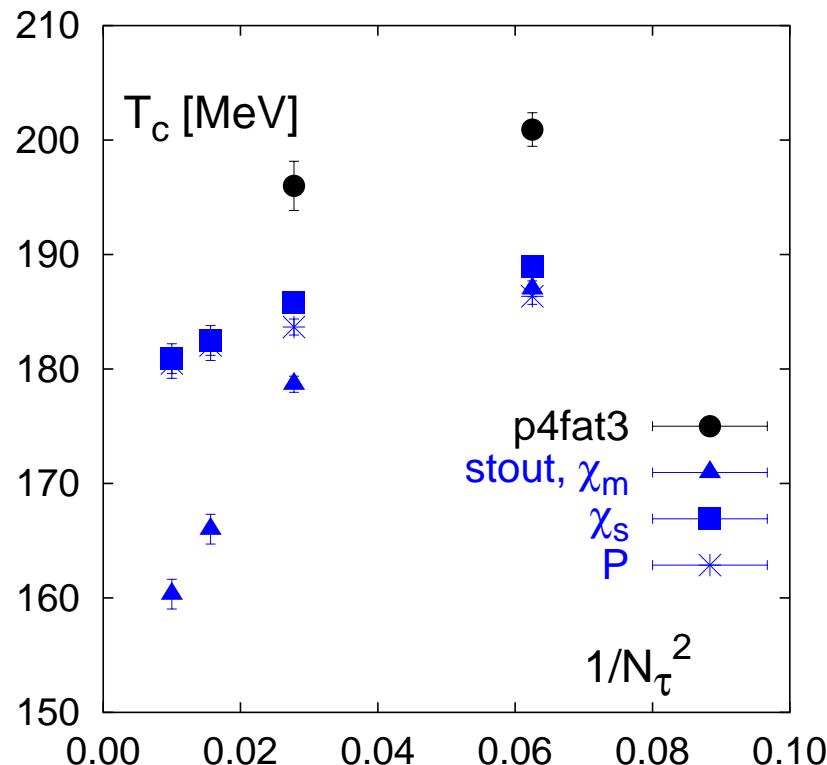


overall scale set with
 $r_0 = 0.469$ fm

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Drastic changes closer to the continuum limit?



- widely different crossover temperatures should also be reflected in the EoS
- current EoS at $T \simeq 150$ MeV:
 $\epsilon \simeq 65$ MeV/fm³ $\simeq \epsilon_0$
($\epsilon_0 \equiv \epsilon$ in uncompressed nuclear matter)

⇒ need to confirm results on EoS closer to the continuum limit

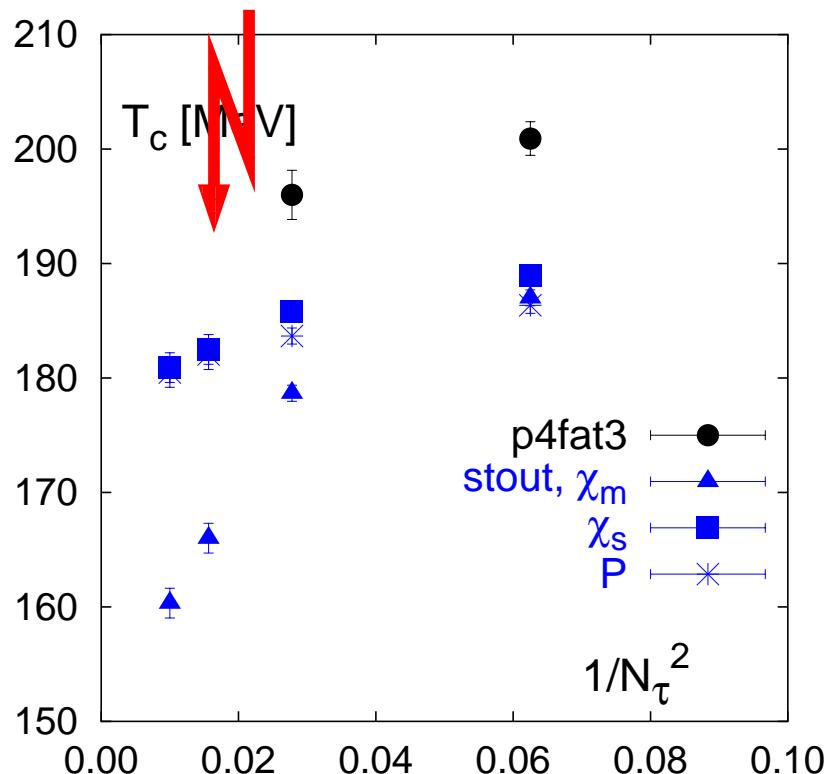
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$N_\tau = 8$: hotQCD collaboration (ongoing)

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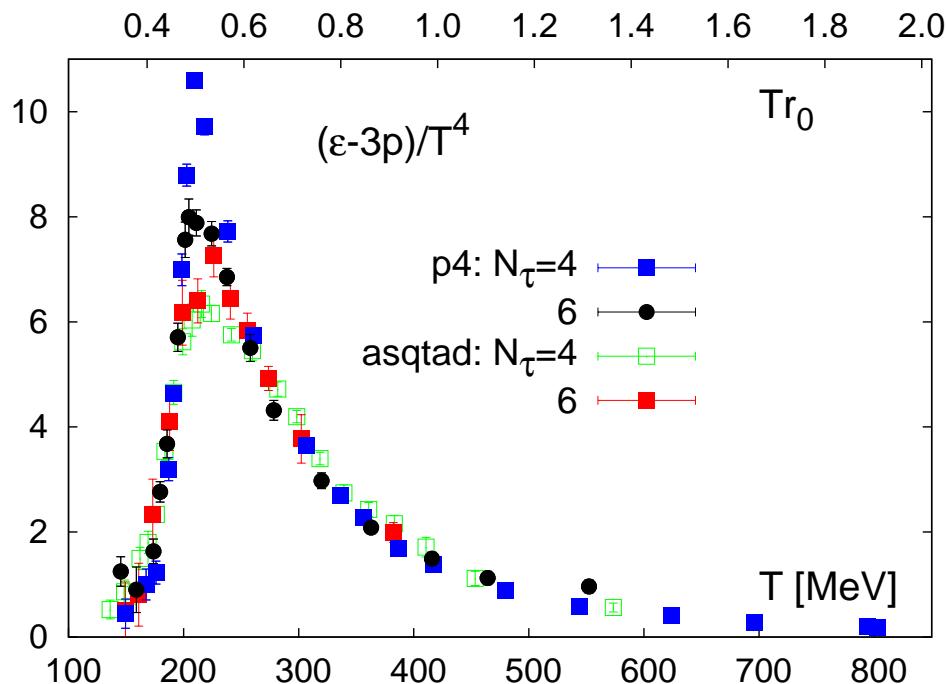
overall scale set with
 $r_0 = 0.469$ fm

$(\epsilon - 3p)/T^4$ on LCP

- requires good control over $T > 0$ observables (action differences, chiral condensates) as well as an
- accurate determination of $T = 0$ scales AND β -functions

LCP: $m_\pi \simeq 210$ MeV, $m_{\bar{s}s} \simeq 665$ MeV

p4 vs. asqtad: overall good agreement



RBC-Bielefeld (preliminary)

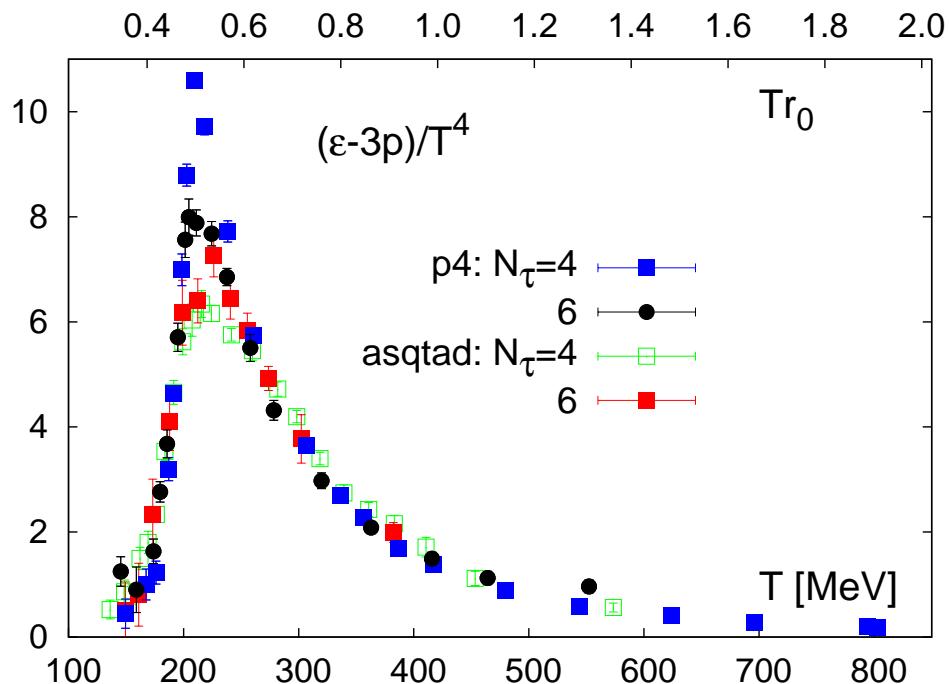
asqtad data:
C. Bernard et al., hep-lat/0611031

Note:
 T -scale is not dependent on
 T_c determination

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C. Bernard et al., hep-lat/0611031

p4 vs. asqtad: overall good agreement

However: We need to...

$T < T_c$ make contact to hadron gas phenomenology

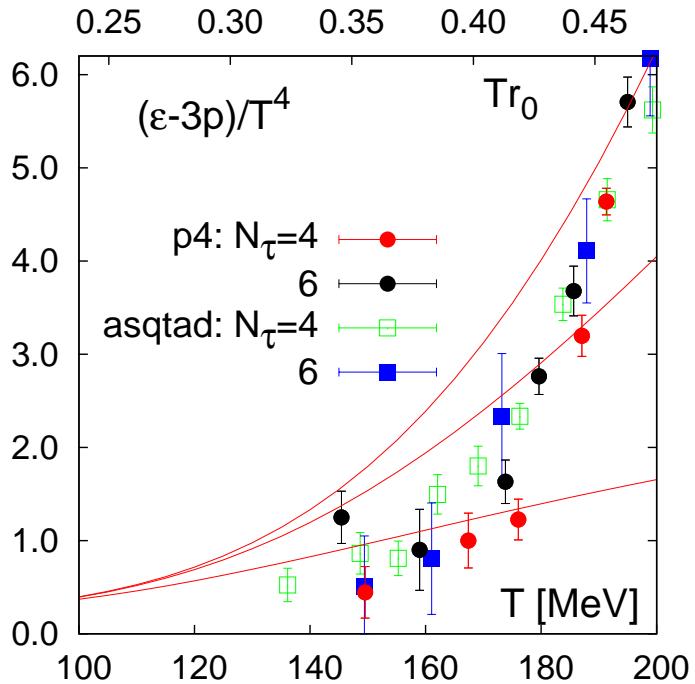
$T < 2T_c$ analyze large deviation from conformal limit ($\epsilon = 3p$)

$T > 2T_c$ make contact to (resummed) perturbation theory

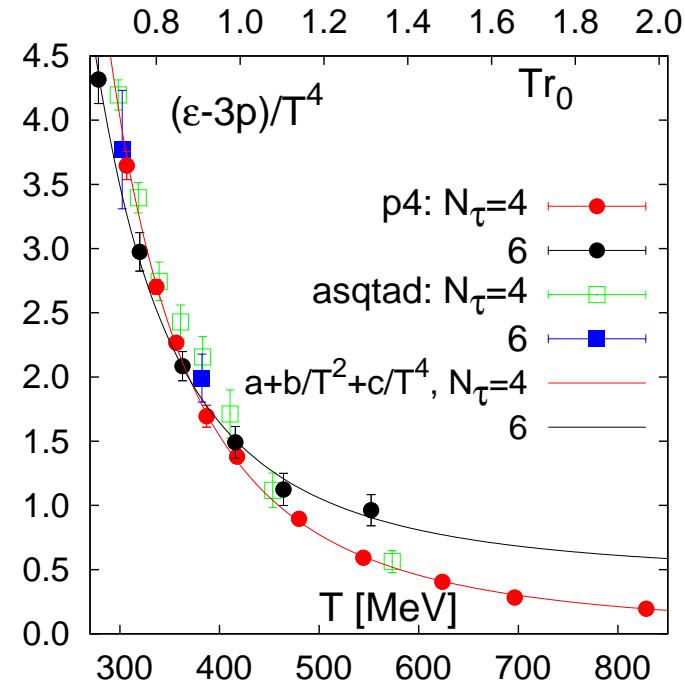
Note:

T -scale is not dependent on T_c determination

EoS: low and high T regime



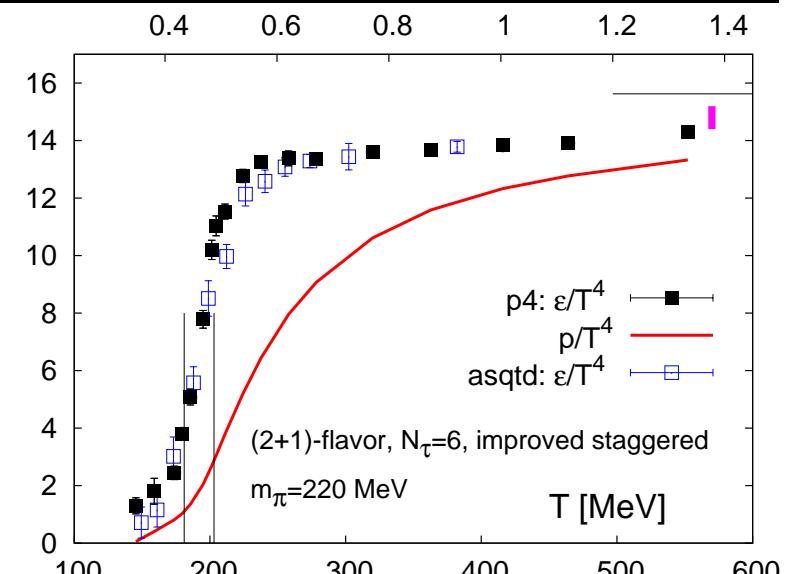
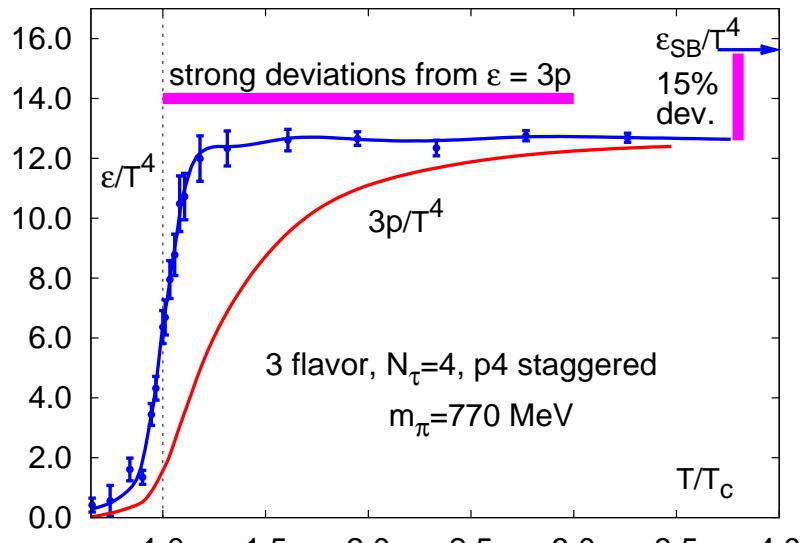
LGT vs resonance gas



LGT vs. pert. theory

- control over low T -regime influence approach to ideal gas limit at high T .
- non-perturbative corrections; resummation approach to perturbative limit
- improved staggered fermions but still on rather coarse lattices:
⇒ need $N_\tau = 8$ (hotQCD collaboration, ongoing) and 10...

$\mu = 0$: QCD Equation of State



- strong deviations from ideal gas behavior ($\epsilon = 3p$) for $T_c \leq T \sim 3T_c$ and even at high T

- smaller lattice cut-off; almost physical light and strange quark masses:
confirms weak quark mass dependence of EoS; closer to SB limit at high-T

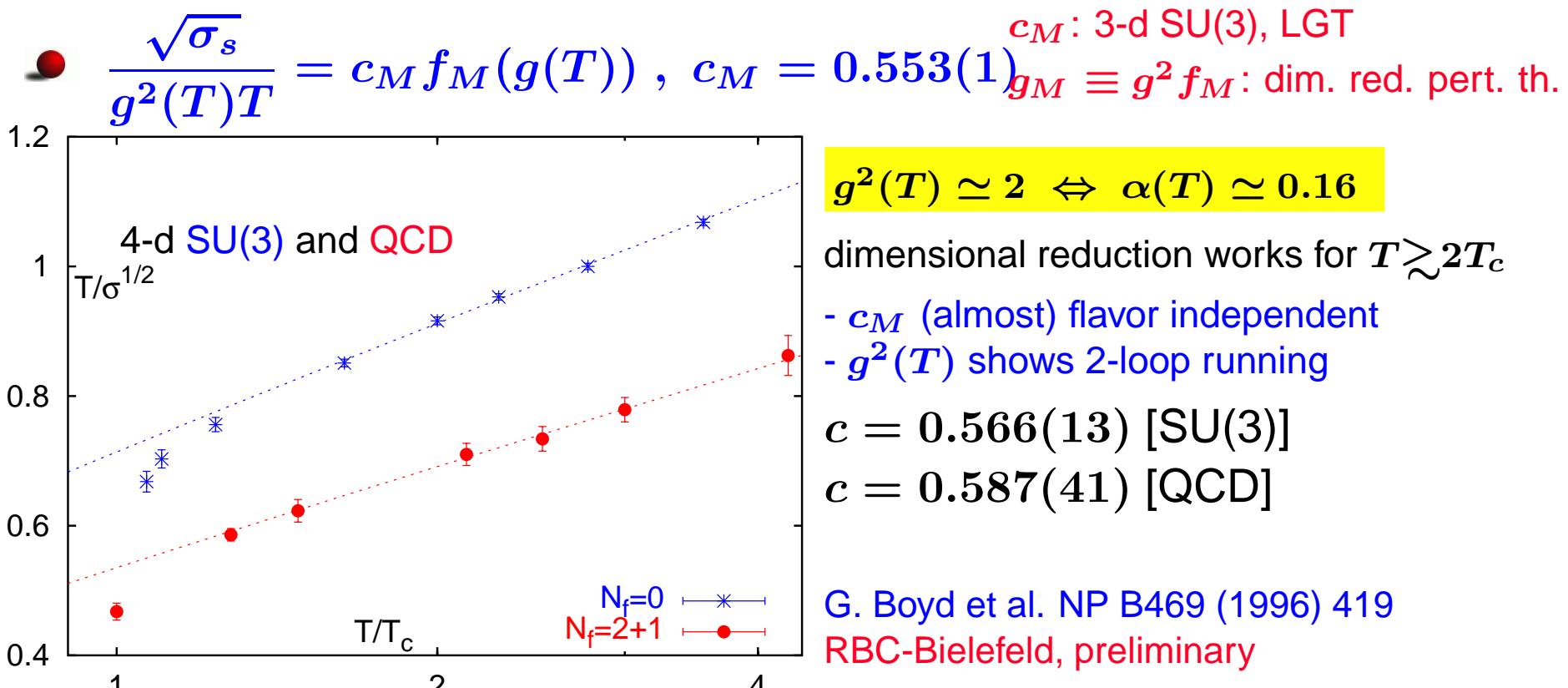
- improved staggered fermions but still on rather coarse lattices:
 \Rightarrow need $N_\tau = 8$, (hotQCD collaboration, ongoing) and 10..

The spatial string tension

Does dimensional reduction work with light quarks?

- Non-perturbative, vanishes in high-T perturbation theory:

$$\sqrt{\sigma_s} = - \lim_{R_x, R_y \rightarrow \infty} \ln \frac{W(R_x, R_y)}{R_x R_y}$$



New projects on BlueGene/L

– 360TFlops at LLNL; 100TFlops at BNL –

Modeling the QCD equation of state closer to the continuum limit

joint project: BNL-RIKEN-Columbia, LANL, LLNL and MILC

collaboration on the Livermore BlueGene/L

- T_c , EoS on $N_\tau = 8$ lattices with light dynamical quarks:
(2+1)-flavor QCD, close to physical m_π/m_K ratio;
exploring the continuum limit: $N_\tau = 4, 6, 8$
analyzing the thermodynamic limit: $V \simeq 500 \text{ fm}^3$

EoS on $32^3 \times 8$ lattices; CPU-time: $\sim (20\text{-}40) \text{ TFlops-years}$
- **FUTURE:** test universal properties, details of χ -symmetry restoration
⇒ bulk thermodynamics with chiral fermions
requires $\mathcal{O}(50)$ more computing resources
⇒ PETAFLOPS computing

QCD Thermodynamics 2005-2009: What do we want to know?

II. search for the critical point at $\mu > 0$:

- **chiral critical point:** does it exist?; location of the chiral critical point;
direct evidence for 1st order regime;
location of the transition line for $\mu > 0$; density fluctuations; $T_c(\mu) \equiv T_{\text{freeze}}$?

Extending the phase diagram to non-vanishing chemical potential

non-zero baryon number density: $\mu > 0$

$$\begin{aligned} Z(\mathbf{V}, \mathbf{T}, \boldsymbol{\mu}) &= \int \mathcal{D}\mathcal{A} \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S_E(\mathbf{V}, \mathbf{T}, \boldsymbol{\mu})} \\ &= \int \mathcal{D}\mathcal{A} \mathcal{D} \det M(\boldsymbol{\mu}) e^{-S_E(\mathbf{V}, \mathbf{T})} \end{aligned}$$

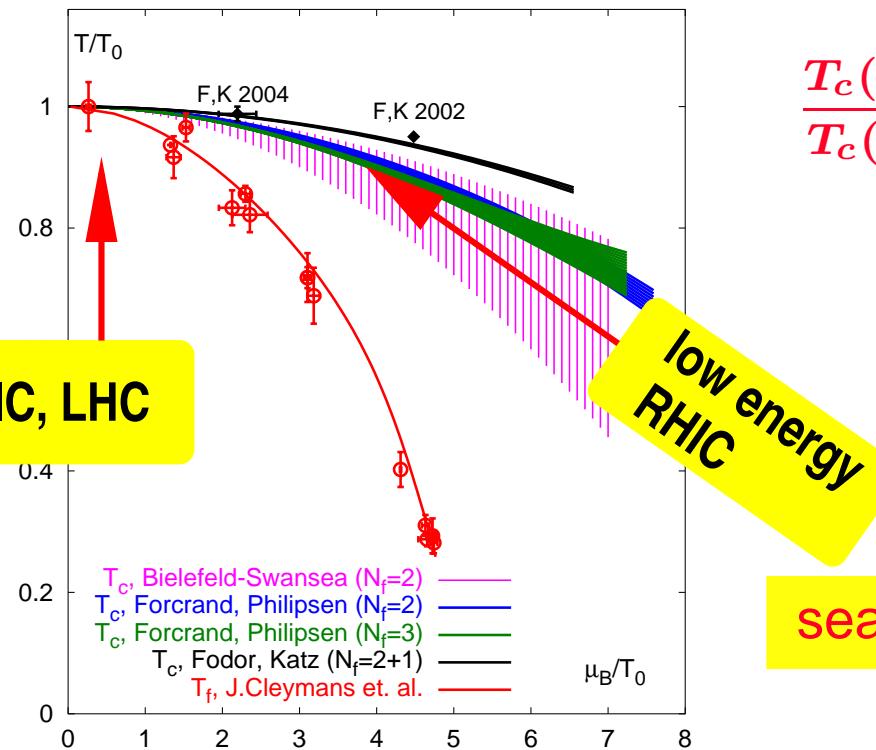
\uparrow complex fermion determinant;
long standing problem

- ⇒ three (partial) solutions for large T , small μ
- exact evaluation of $\det M$: works well on small lattices; requires reweighting
[Z. Fodor, S.D. Katz, JHEP 0203 \(2002\) 014](#)
 - Taylor expansion around $\mu = 0$: works well for small μ ; requires reweighting
[C. R. Allton et al. \(Bielefeld-Swansea\), Phys. Rev. D66 \(2002\) 074507](#)
 - imaginary chemical potential: works well for small μ ; requires analytic continuation
[Ph. deForcrand, O. Philipsen, Nucl. Phys. B642 \(2002\) 290](#)

Extending the phase diagram to non-vanishing chemical potential

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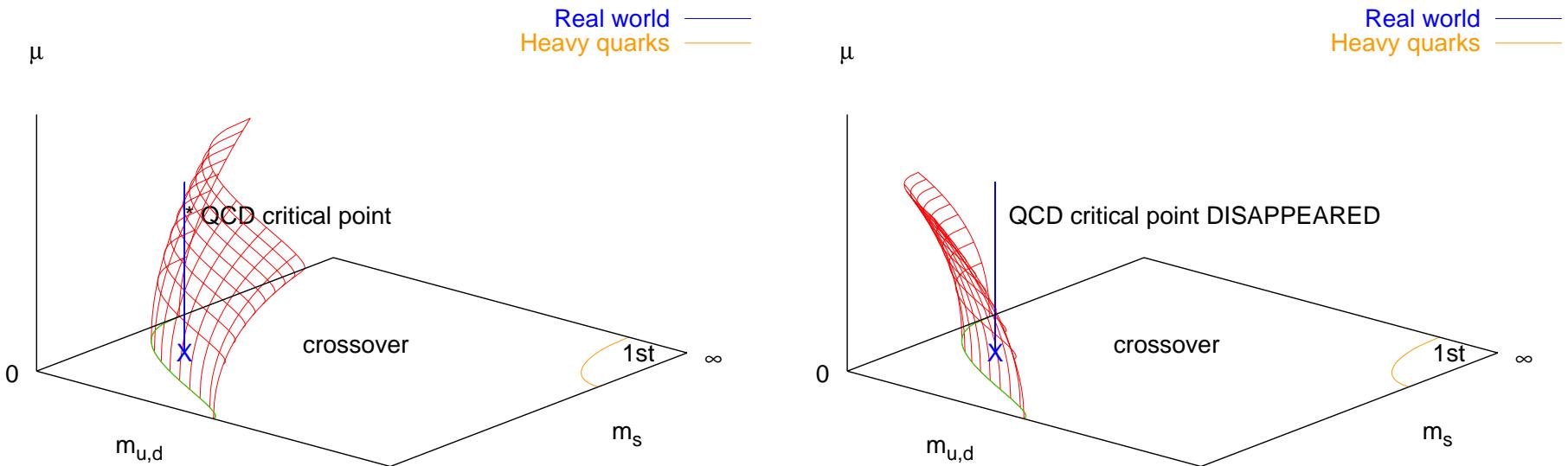


$$\frac{T_c(\mu)}{T_c(0)} : \begin{aligned} &1 - 0.0056(4)(\mu_B/T)^2 \quad \text{deForcrand, Philipsen (imag. } \mu \text{)} \\ &1 - 0.0078(38)(\mu_B/T)^2 \quad \text{Bielefeld-Swansea} \\ &(\mathcal{O}(\mu^2) \text{ reweighting}) \end{aligned}$$

search for critical point

The critical surface:

Does it bend in the 'wrong' direction?



Ph. de Forcrand and O. Philipsen, JHEP 01 (2007) 077

Need to understand the QCD phase diagram in the $m_{u,d}$ - m_s - μ_q space

- determine phase boundary in the $m_{u,d}$ - m_s plane
- determine curvature of the boundary line at $\mu_q \simeq 0$; Taylor expansion
- study fluctuations and correlations of e.g. baryon number, charge, strangeness

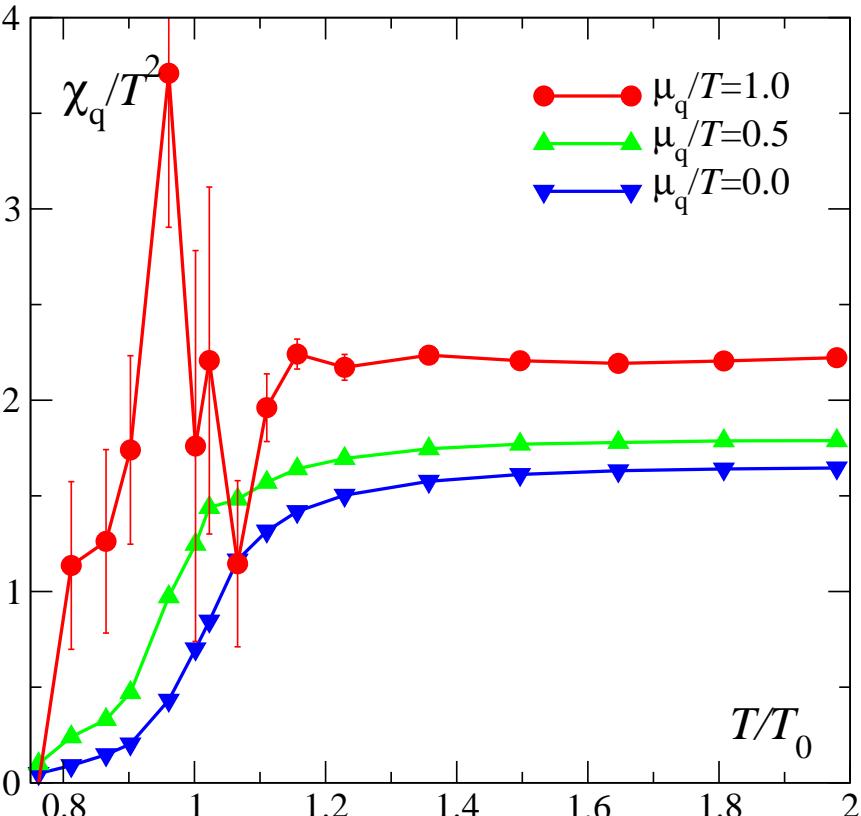
Fluctuations of the baryon, charge and strangeness number density ($\mu \geq 0$)

baryon number density fluctuations:

(Bielefeld-Swansea, PRD68 (2003) 014507)

$$\mu \geq 0, n_f = 2$$

$$m_\pi \simeq 770 \text{ MeV}$$



$$\frac{\chi_q}{T^2} = \left(\frac{d^2}{d(\mu/T)^2} \frac{p}{T^4} \right)_{T \text{ fixed}}$$
$$= \frac{9}{V} (\langle N_B^2 \rangle - \langle N_B \rangle^2)$$

susceptibilities

to be studied in event-by-event fluctuations

Taylor exp. in μ_q, μ_s :

$$\chi_q/T^2 = 2c_{20} + 12c_{40}(\mu_q/T)^2$$

$$\chi_s/T^2 = 2c_{02} + 2c_{22}(\mu_q/T)^2$$

recent papers:

V. Koch, E.M. Majumder, J. Randrup, nucl-th/0505052

S. Ejiri, FK, K. Redlich, hep-ph/05090521

R.V. Gavai, S. Gupta, hep-lat/0510044

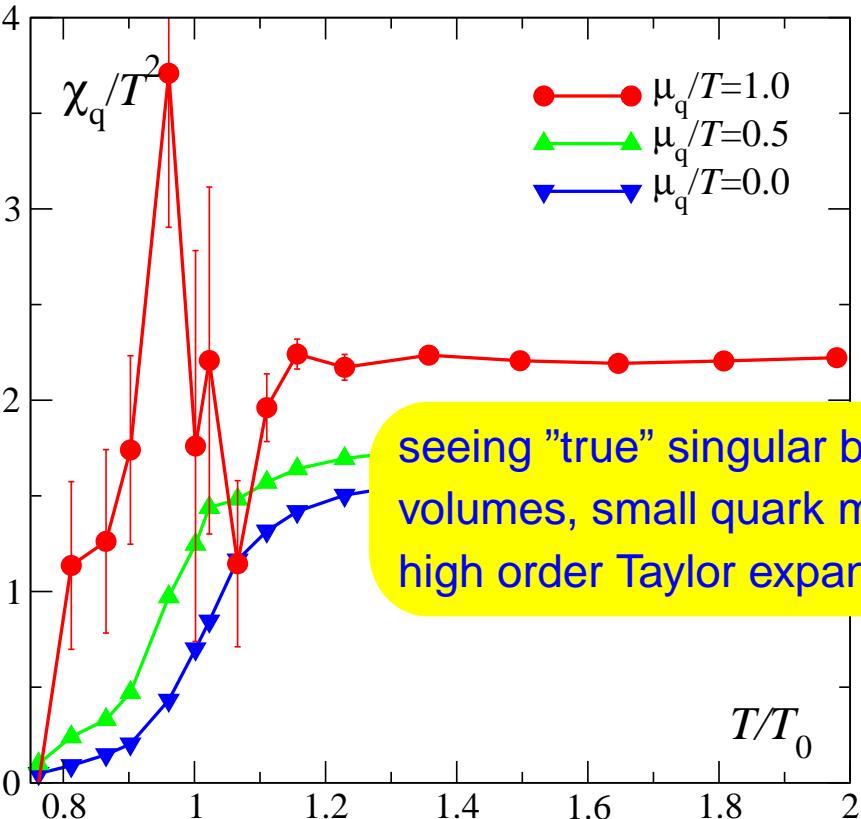
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seeing "true" singular behaviour requires large
volumes, small quark masses and/or
high order Taylor expansions \Rightarrow RBC-BI

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susceptibilities

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Taylor exp. in μ_q, μ_s :

$$\sim 1/T^2 - 2c_{20} + 12c_{40}(\mu_q/T)^2$$

$$+ 2c_{22}(\mu_q/T)^2$$

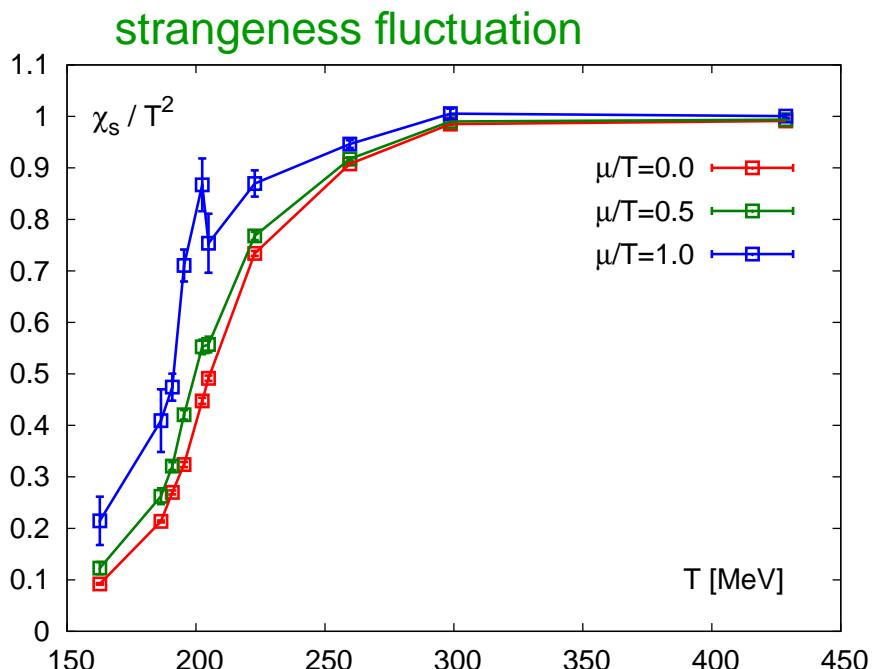
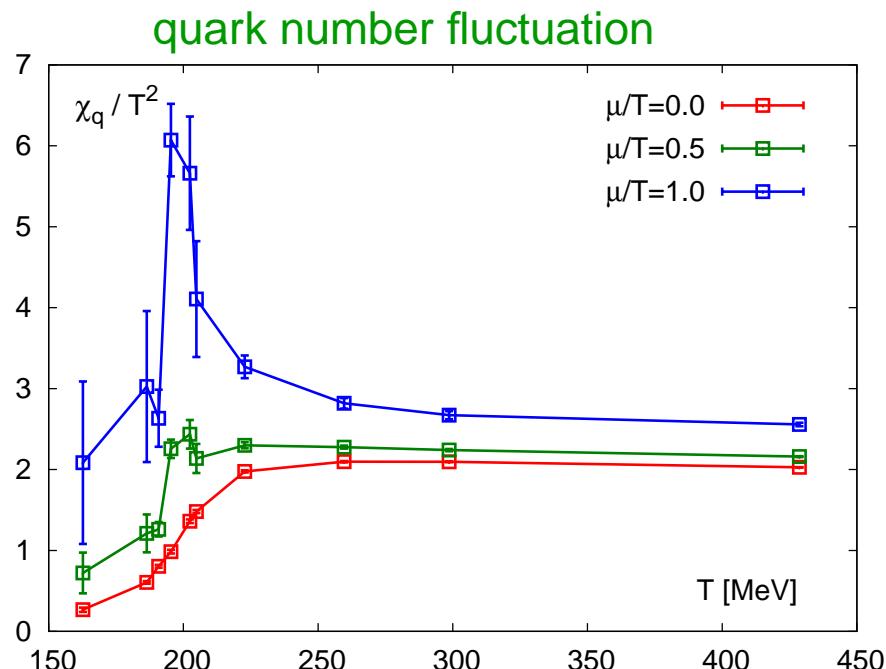
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Fluctuations of baryon number and strangeness in (2+1)-flavor QCD

RBC-Bielefeld, in preparation

$$\chi_q/T^2 = 2c_{20} + 12c_{40}(\mu_q/T)^2$$

$$\chi_s/T^2 = 2c_{02} + 2c_{22}(\mu_q/T)^2$$



⇒ large quark number fluctuations

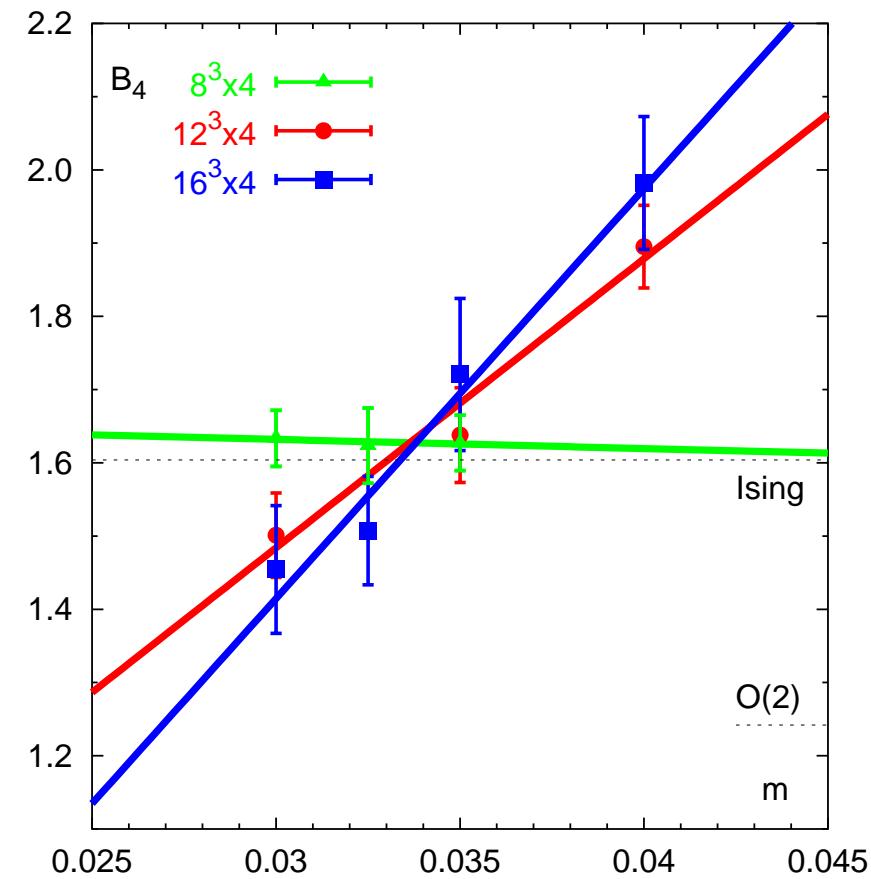
⇒ enhance strangeness fluctuations (factor ~ 2)

The transition line at $\mu_q = 0$

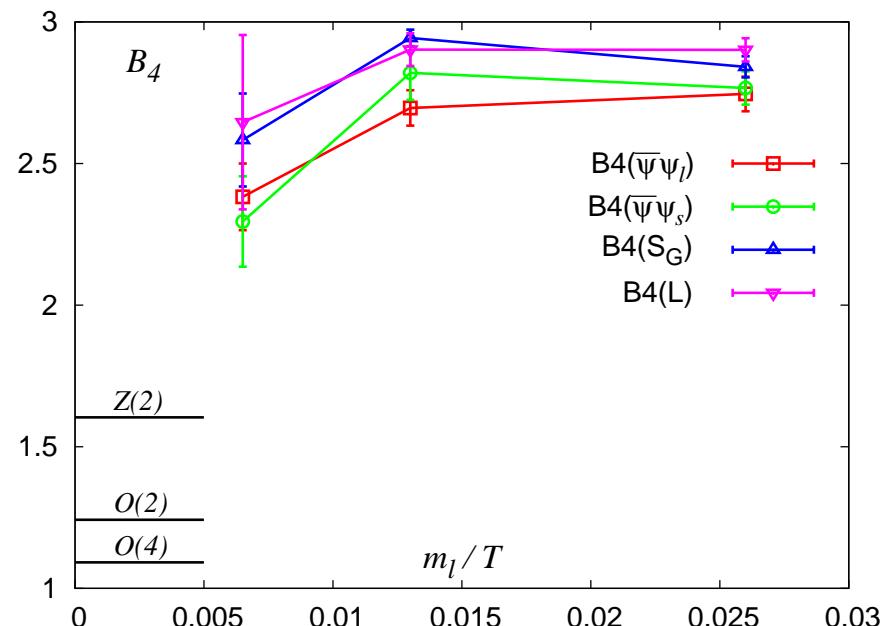
Binder cumulants: $B_4 = \frac{\langle (\delta\psi\bar{\psi})^4 \rangle}{\langle (\delta\psi\bar{\psi})^2 \rangle^2}$

RBC-Bielefeld, 2007/08

standard staggered: 3-flavor QCD



p4: (2+1)-flavor QCD



\Rightarrow improved actions lead to smaller critical mass

The transition line at $\mu_q = 0$ and its extension to $\mu_q > 0$

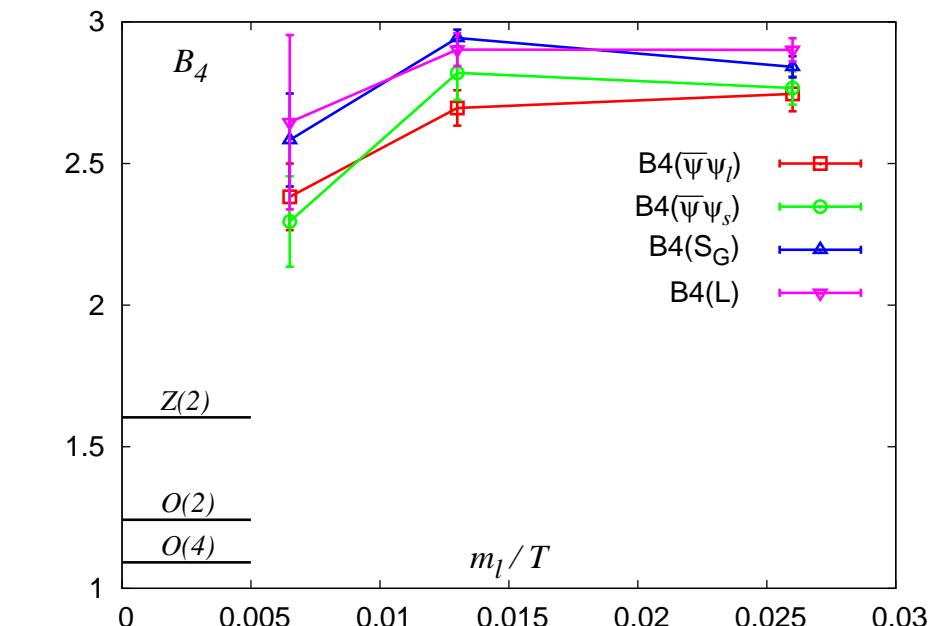
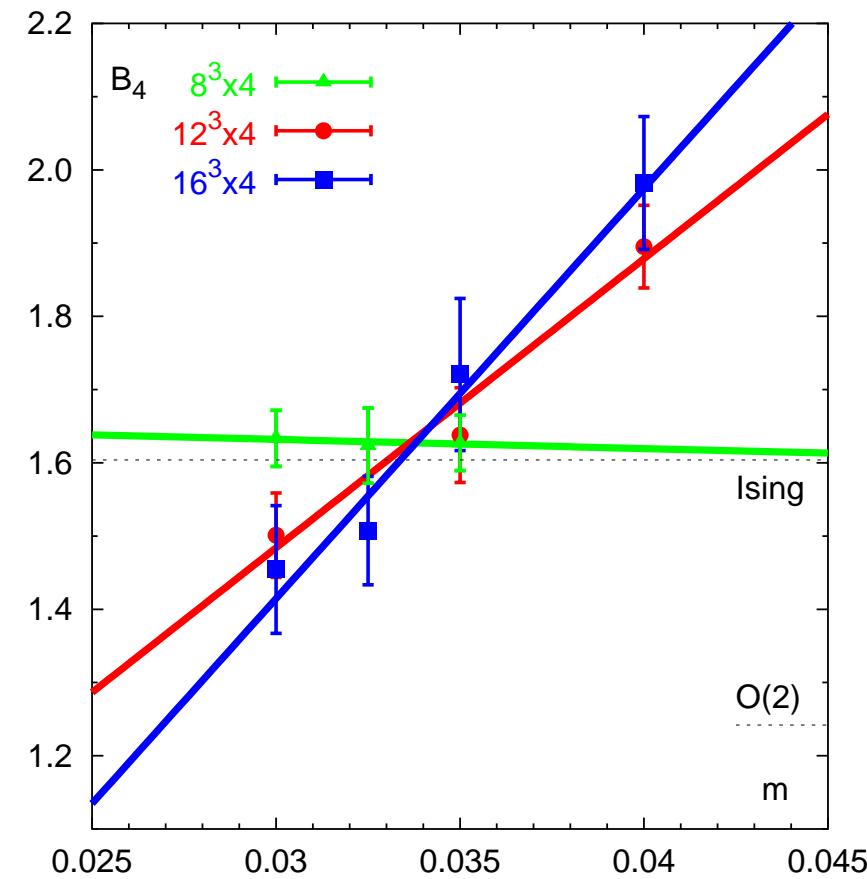
Binder cumulants:

$$B_4 = \frac{\langle (\delta\psi\bar{\psi})^4 \rangle}{\langle (\delta\psi\bar{\psi})^2 \rangle^2} = B_{4,0} + B_{4,2} \left(\frac{\mu_q}{T}\right)^2$$

standard staggered: 3-flavor QCD

RBC-Bielefeld, 2007/08

p4: (2+1)-flavor QCD



\Rightarrow improved actions lead to smaller critical mass

Outlook: projects on future machines...

...towards thermodynamics on Petaflops computers

(extension of (exploratory) studies on current Teraflops computers)

- Thermodynamics closer to the continuum limit
- Thermodynamics of QCD with chiral fermion formulations
- finite density QCD:
aim at definite answer on the existence and location of a critical point;
try to reach lower temperatures around $T \sim 0.5 T_c$
- In-medium properties of light quark bound states:
QCD with light, dynamical quarks on fine lattices become possible;
mass shifts and modification of widths below T_c
- transport properties:
calculation of "gluonic correlator" (energy momentum tensor) should become possible; spectral functions in the $\omega \rightarrow 0$ limit may become accessible
(dilepton rates; heavy quark diffusion coefficient,...)